Integrable models, degenerate horizons and AdS_2 black holes

J. Cruz,[a] A. Fabbri,[b] D. J. Navarro[a], J. Navarro-Salas[a] and P. Navarro[a]

[a]Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Facultad de Física, Universidad de Valencia, Burjassot-46100, Valencia, Spain.

[b]Dipartimento di Fisica dell'Università di Bologna and INFN sezione di Bologna, Via Irnerio 46, 40126 Bologna, Italy.

The near extremal Reissner-Nordström black holes in arbitrary dimensions can be modeled by the Jackiw-Teitelboim (JT) theory. The asymptotic Virasoro symmetry of the corresponding JT model exactly reproduces, via Cardy's formula, the deviation of the Bekenstein-Hawking entropy of the Reissner-Nordström black holes from extremality. We also comment how can we extend this approach to investigate the evaporation process.

The understanding of the thermodynamical properties and the dynamical evolution of black holes is a crucial aspect in the formulation of a consistent theory of quantum gravity. Recent improvements of string theory have been able to provide a microscopical structure which accounts for the Bekenstein-Hawking entropy and gives a mechanism for the evaporation process of a black hole (see the review [1]). More recently, the Bekenstein-Hawking entropy of three-dimensional BTZ black holes has been derived using only symmetry properties of three-dimensional gravity [2] (in this approach there are some technical subtleties [3]). This suggests that the entropy formula may be insensitive to the details of the quantum theory. Further results in this direction can be found in [4,5].

An extremal 5d Reissner-Nordström black hole was the first case in which it was possible to identify the microscopic degrees of freedom responsible for the Bekenstein-Hawking entropy [6]. The extremality condition was crucial since it corresponds, although this link is not general, to a BPS state. The deviation of the near-extremal entropy [7] is also related to the BPS bound. In this work we want to point out that, since the extremality condition is also related to the existence of degenerate horizons and hence to two-dimensional Anti-de Sitter geometry, a realization of the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence [8,9] can be used to explain the near-extremal Bekenstein-Hawking entropy of Reissner-Nordström black holes in arbitrary dimensions. Moreover we argue that the two-dimensional effective theory which codifies the dynamics near extremality could also be used to provide a picture of the evaporation process.

Let us start our analysis with the Einstein-Maxwell action in $(n+2)$ dimensions

$$\begin{aligned} I^{(n+2)} &= \frac{1}{16\pi l^n}\int d^{n+2}x\sqrt{-g^{(n+2)}}\Big(R^{(n+2)} \\ &+ (F^{(n+2)})^2\Big)\,, \end{aligned} \tag{1}$$

where l^n is Newton's constant $G^{(n+2)}$. Imposing spherical symmetry on the gauge field and decomposing the metric in the form

$$\begin{aligned} ds^2_{(n+2)} &= \frac{8(n-1)}{n^2}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{n}{n-1}} ds^2_{(2)} \\ &+ l^2\left(\frac{8(n-1)}{n}\phi\right)^{\frac{2}{n}} d\Omega^2_{(n)}\,, \end{aligned} \tag{2}$$

the above action reduces to

$$I^{(n+2)} = \frac{1}{2G}\int d^2x\sqrt{-g}(R\phi + l^{-2}V(\phi))\,, \tag{3}$$

where

$$G = \frac{n\pi}{(n-1)\mathcal{V}^{(n)}}\,, \tag{4}$$

being $\mathcal{V}^{(n)}$ the area of the unit S^n sphere and

$$\begin{aligned} V(\phi) &= (n-1)\left(\frac{8(n-1)}{n}\phi\right)^{\frac{-1}{n}} \\ &- \frac{2(n-1)^2 l^2 q^2}{n}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{1-2n}{n}} . \end{aligned} \tag{5}$$

In terms of the two-dimensional metric the Reissner-Nordström black hole solutions take the form

$$\begin{aligned} ds^2_{(2)} &= -U(\phi)dt^2 + U(\phi)^{-1}dx^2 \,, \\ U(\phi) &= J(\phi) - 2Glm \,, \\ \phi &= \frac{x}{l} \,, \end{aligned} \tag{6}$$

where $J(\phi) = \int_0^\phi d\tilde{\phi} V(\tilde{\phi})$. The horizons of the black hole correspond to

$$J(\phi) = 2Glm \,, \tag{7}$$

and the degenerate horizons emerge when

$$J'(\phi_0) = 0 \,. \tag{8}$$

The idea now is to perform a perturbation around the degenerate radius of coincident horizons [10]

$$m = m_0(1 + k\alpha^2) \,, \tag{9}$$

$$t = \frac{\tilde{t}}{\alpha} \,, \tag{10}$$

$$x = x_0 + \alpha\tilde{x} \,, \tag{11}$$

$$\phi = \phi_0 + \alpha\tilde{\phi} \,, \tag{12}$$

where $J(\phi_0) = 2Glm_0$. We then have

$$\begin{aligned} ds^2 &= -\tilde{U}(r)d\tilde{t}^2 + \frac{d\tilde{x}^2}{\tilde{U}(r)} + r_0^2 d\Omega^2 + \mathcal{O}(\alpha) \,, \\ \tilde{U}(r) &= -\frac{\tilde{R}_0}{2}\tilde{x}^2 - km_0 l \,, \end{aligned} \tag{13}$$

where

$$\tilde{R}_0 = -\frac{J''(\phi_0)}{l^2} \,. \tag{14}$$

The two-dimensional effective action can also be expanded

$$\begin{aligned} I^{(n+2)} &= \frac{\alpha}{2G}\int d^2x\sqrt{-g}(R\tilde{\phi} \\ &+ l^{-2}V'(\phi_0)\tilde{\phi}) + \mathcal{O}(\alpha^2) \,, \end{aligned} \tag{15}$$

and the leading order term is just the Jackiw-Teitelboim model. This theory possesses an asymptotic set of symmetries [11,9] which preserve the asymptotic form of the metric

$$g_{\tilde{t}\tilde{t}} = \frac{\tilde{R}_0}{2}\tilde{x}^2 + \gamma_{\tilde{t}\tilde{t}} + \dots \,, \tag{16}$$

$$g_{\tilde{t}\tilde{x}} = \frac{\gamma_{\tilde{t}\tilde{x}}}{\tilde{x}^3} + \dots \,, \tag{17}$$

$$g_{\tilde{x}\tilde{x}} = -\frac{2}{\tilde{R}_0}\frac{1}{\tilde{x}^2} + \frac{\gamma_{\tilde{x}\tilde{x}}}{\tilde{x}^4} + \dots \,. \tag{18}$$

The infinitesimal diffeomorphisms $\zeta^a(\tilde{x},\tilde{t})$ preserving the above boundary conditions are

$$\zeta^{\tilde{t}} = \epsilon(\tilde{t}) - \frac{2}{\tilde{R}_0^2\tilde{x}^2}\epsilon(\tilde{t}) + \mathcal{O}\left(\frac{1}{\tilde{x}^4}\right) \,, \tag{19}$$

$$\zeta^{\tilde{x}} = -\tilde{x}\epsilon'(\tilde{t}) + \mathcal{O}\left(\frac{1}{\tilde{x}}\right) \,, \tag{20}$$

where $\epsilon(\tilde{t})$ is an arbitrary function. The remaining terms in the above expansion represent pure gauge transformations. The following expression

$$\Theta_{\tilde{t}\tilde{t}} = \kappa\left(\gamma_{\tilde{t}\tilde{t}} - \frac{1}{2}\left(\frac{\tilde{R}_0}{2}\right)^2\gamma_{\tilde{x}\tilde{x}}\right) \,, \tag{21}$$

where κ is a model-dependent constant, is the unique "gauge invariant" quantity and it transforms according to the rule

$$\delta_\epsilon\Theta_{\tilde{t}\tilde{t}} = \epsilon(\tilde{t})\Theta'_{\tilde{t}\tilde{t}} + 2\Theta_{\tilde{t}\tilde{t}}\epsilon'(\tilde{t}) - \frac{2\kappa}{\tilde{R}_0}\epsilon'''(\tilde{t}) \,. \tag{22}$$

Since Anti-de Sitter space has a natural periodicity in $\tilde{t}$ we shall assume that it varies in the interval $0 \le \tilde{t} \le 2\pi\beta$. Defining the Fourier components of $\Theta_{\tilde{t}\tilde{t}}$ as

$$L_n^R = \frac{1}{2\pi\beta}\int_0^{2\pi\beta} d\tilde{t}\Theta_{\tilde{t}\tilde{t}}\beta e^{in\frac{\tilde{t}}{\beta}} \,, \tag{23}$$

the Poisson algebra can be computed from

$$\{L_n, L_m\} = \delta_{\epsilon_m} L_n \,, \tag{24}$$

where $\epsilon_m = \beta e^{im\frac{\tilde{t}}{\beta}}$, and the result is a Virasoro algebra with central charge

$$c = -\frac{24}{\tilde{R}_0\beta}\kappa \,. \tag{25}$$

The coefficient κ is determined by the effective Lagrangian near extremality and can be computed using Hamiltonian methods. We find that $\kappa = \frac{\alpha}{lG}$, so

$$c = -\frac{24\alpha}{lG\tilde{R}_0\beta} \,. \tag{26}$$

One can also work out the value of L_0

$$L_0 = m_0\kappa\alpha\beta \,. \tag{27}$$

We must remark here that in the Fourier decomposition the parameter $\tilde{t}$ plays the role of a null coordinate of a 2d CFT. This is a consequence of the general fact [12] that the conformal group in the AdS_2/CFT_1 correspondence can be regarded as one chiral component of the two-dimensional conformal group. With these values the product cL_0 which appears in the Cardy formula is insensitive to the arbitrary parameters introduced so far and only depends on physical quantities. We have

$$2\pi\sqrt{\frac{cL_0}{6}} = 2\pi\sqrt{\frac{4\Delta m}{-\tilde{R}_0 lG}} \,, \tag{28}$$

and taking into account that

$$\tilde{R}_0 = \frac{16(n-1)^3}{-l^2n^2}\left(\frac{n}{2(n-1)l^2q^2}\right)^{\frac{1+n}{2(n-1)}} \,, \tag{29}$$

the above expression exactly coincides with the deviation ΔS of the Bekenstein-Hawking entropy

$$S^{BH} = \frac{\mathcal{V}^{(n)}r^n}{4l^n} \,, \tag{30}$$

from the extremal case. Therefore, two-dimensional gravity theories codify adequately the effective dynamics in such a way that they allow to provide a microscopic derivation of the near-extremal black hole entropy. So it is also natural to investigate whether the Jackiw-Teitelboim model can provide a mechanism for the evaporation process of a Reissner-Nordström black hole near extremality.

We shall now mention briefly how one can investigate the evaporation of Reissner-Nordström black holes using the approximation (15). The Jackiw-Teitelboim theory is an integrable 2d dilaton gravity model [13] with general solution given by the expression [13,14] (here we introduce $\lambda^2 = \frac{V'(\phi_0)}{4l^2}$)

$$ds^2 = -\frac{\partial_+A_+\partial_-A_-}{(1+\frac{\lambda^2}{2}A_+A_-)^2} \,, \tag{31}$$

$$\begin{aligned}\phi &= -\frac{1}{2}\left(\frac{\partial_+\tilde{a}_+}{\partial_+A_+} + \frac{\partial_-\tilde{a}_-}{\partial_-A_-}\right) \\ &+ \frac{\lambda^2}{2}\frac{A_+\tilde{a}_- + A_-\tilde{a}_+}{1+\frac{\lambda^2}{2}A_+A_-} \,,\end{aligned} \tag{32}$$

where the chiral functions $A_\pm$, $a_\pm$ verify the constraint equations

$$\begin{aligned}T^f_{\pm\pm} &= \pm\partial^2_\pm\left(\frac{\partial_\pm a_\pm}{\partial_\pm A_\pm}\right) \\ &\mp \frac{\partial^2_\pm A_\pm}{\partial_\pm A_\pm}\,\partial\pm\left(\frac{\partial_\pm a_\pm}{\partial_\pm A_\pm}\right) .\end{aligned} \tag{33}$$

Working in the conformal gauge we can match along the null line $x^+ = x^+_0$ the solution $\phi = 0$ ($a_\pm = 0$), representing an extremal black hole, with the solution $\phi \neq 0$

$$\phi = C\frac{\frac{\lambda^2}{2}(x^+\Delta^- + x^-\Delta^+) - 1 + \frac{\lambda^2}{2}x^+x^-}{1+\frac{\lambda^2}{2}x^+x^-} \,, \tag{34}$$

where $\Delta^+ = -x^+_0$, $\Delta^- = \frac{2}{\lambda^2 x^+_0}$ and

$$ds^2 = \frac{-dx^+dx^-}{(1+\frac{\lambda^2}{2}x^+x^-)^2} \,, \tag{35}$$

everywhere. The above dynamics is originated by the shock wave

$$T^f_{++} = \frac{C}{x^+_0}\delta(x^+ - x^+_0) \tag{36}$$

and therefore it seems natural to associate the solution for $x^+ > x^+_0$ to a near extremal hole.

The solvability of the classical model when matter is coupled in a conformal way can also be extended to the one-loop theory. The effective action is given by

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left(R\tilde{\phi} + 4\lambda^2\tilde{\phi}\right.$$

$$- \quad \sum_{i=1}^{N} \frac{1}{2} |\nabla f_i|^2 \Big) - \frac{N}{96\pi} \int \sqrt{-g} R \Box^{-1} R$$
$$+ \quad \frac{N}{12\pi} \int d^2x \sqrt{-g} \lambda^2 , \qquad (37)$$

where the N fields f_i model the matter degrees of freedom.

The unconstrained equations of motion remain as the classical ones, but the constrained equations get modified according to

$$T^f_{\pm\pm} - \tfrac{N}{12} t_\pm =$$
$$\pm \tfrac{1}{2} \partial^2_\pm \left(\frac{\partial_\pm a_\pm}{\partial_\pm A_\pm} \right) \mp \tfrac{1}{2} \frac{\partial^2_\pm A_\pm}{\partial_\pm A_\pm} \partial \pm \left(\frac{\partial_\pm a_\pm}{\partial_\pm A_\pm} \right)$$
$$+ \tfrac{N}{12} \left[\tfrac{1}{4} \left(\frac{\partial^2_\pm A_\pm}{\partial_\pm A_\pm} \right)^2 - \tfrac{1}{2} \partial_\pm \left(\frac{\partial^2_\pm A_\pm}{\partial_\pm A_\pm} \right) \right] \quad , \quad (38)$$

where the functions $t_\pm(x^\pm)$ are related with the boundary conditions of the theory. The crucial point now is to choose the proper physical boundary functions $t_\pm$. This question will be investigated in a further work, but we want to stress that, if the JT model has been able to explain the deviation of the near extremal entropy one can also expect that a rigorous study would offer an approximate but useful picture of the evaporation process of near-extremal Reissner-Nordström black holes.

Acknowledgements

This research has been partially supported by the CICYT and DGICYT, Spain. J. Cruz acknowledges the Generalitat Valenciana for a FPI fellowship. D. J. Navarro thanks the Ministerio de Educación y Cultura for a FPI fellowship. P. Navarro acknowledges the Ministerio de Educación y Cultura for a FPU fellowship.

REFERENCES

1. J.M.Maldacena, *"Black holes in string theory"*, hep-th/9607235.
2. A.Strominger, *JHEP* **2** (1998) 9, hep-th/9712251.
3. S.Carlip, *Class. Quant. Grav.* **15** (1998) 3609, hep-th/9806026.
4. S.Carlip, *Phys. Rev. Lett.* **82** (1999) 2828, hep-th/9812013; *Class. Quant. Grav.* **16** (1999) 3327, hep-th/9906126.
5. S.N.Solodukhin, *Phys. Lett.* **B454** (1999) 213, hep-th/9812056.
6. A.Strominger and C.Vafa, *Phys. Lett.* **B379** (1996) 99, hep-th/9601029.
7. C.G.Callan and J.M.Maldacena, *Nucl. Phys.* **B472** (1996) 591, hep-th/9602043
8. J.Navarro-Salas and P.Navarro, *"AdS_2/CFT_1 correspondence and near-extremal black hole entropy"*, hep-th/9910076.
9. D.J.Navarro, J.Navarro-Salas and P.Navarro, *"Holography, degenerate horizons and entropy"*, hep-th/9911091.
10. J.Cruz, A.Fabbri, D.J.Navarro and J.Navarro-Salas, *"Integrable models and degenerate horizons in two-dimensional gravity"*, hep-th/9906187, to appear in *Phys. Rev.* **D**.
11. M. Cadoni and S. Mingemi, *Phys. Rev.* **D59** (1999) 081501, hep-th/9810251; *Nucl. Phys.* **B557** (1999) 165, hep-th/9902040.
12. A.Strominger, *JHEP* **1** (1999) 7, hep-th/9809027.
13. A. T. Filippov, *Mod. Phys. Lett.* **A11** (1996) 1691, hep-th/9605008; *Int. J. Mod. Phys.* **A12** (1997) 13, hep-th/9612058.
14. J. Cruz, J. M. Izquierdo, D. J. Navarro and J. Navarro-Salas, *Phys. Rev.* **D58** (1998) 044010, hep-th/9704168.